Assignments

Teaching A First Course in Astronomy and Astrophysics

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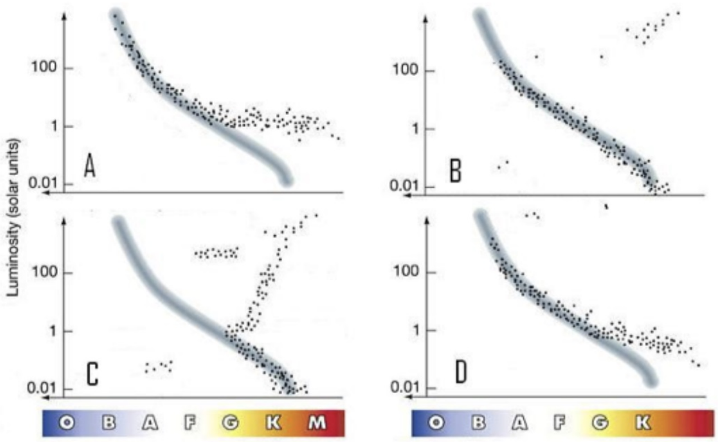
1. **Mass determination I**
The maximal radial velocities measured for the two components of a spectroscopic binary are 100 and 200 km/s, with an orbital period of 2 days. The orbits are circular.
(a) Find the mass ratio of two stars.
(b) Calculate the value of $M \sin^3 i$ for each star.
(c) Find the masses of the two stars if $\sin^3 i$ has its mean value.

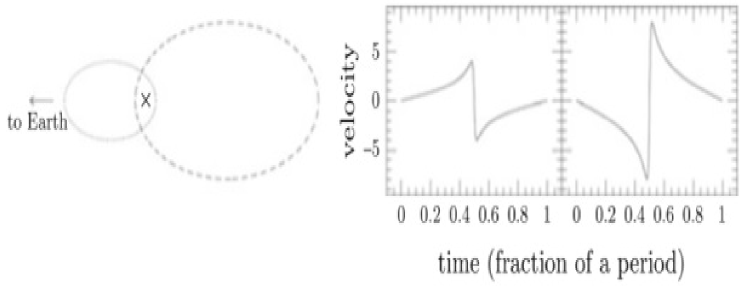
2. **Mass determination II**
In an eclipsing spectroscopic binary, the maximal radial velocities measured for the two components are 20 and 5 km/s. The orbit is circular and orbital period is 5 yr. It takes 0.3 day from the start of the eclipse to the main minimum, which then lasts 1 day.
(a) Find the mass of each star.
(b) Find the radius of each star. Suppose the radii come out to be $r_1 = 2.0R_S$ and $r_2 = 0.46R_S$ (where, R_S is the solar radius). What can you conclude about the inclination angle ?

3. **HR diagrams of clusters**
Below are some sample HR diagrams for star clusters of different ages. Given that O, B, A, F, G, K, and M stars live approximately 3 million, 20 million, 300 million, 2 billion, 10 billion, 50 billion, and 200 billion years respectively on the main sequence, answer the questions below.

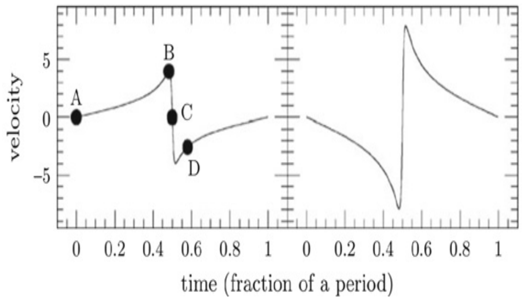


(a) Which star cluster (A, B, C or D) is the oldest and why?
(b) Approximately how old is the oldest star cluster you selected in (a) ?
(c) Approximately how old is the second oldest cluster?
(d) Which cluster is the youngest?
(e) Which clusters have stars that are still forming? Briefly explain how you know.
(f) Which clusters have stars that are not supporting themselves through nuclear fusion? Briefly explain for each of the clusters you select, which stars in the cluster are the ones NOT doing nuclear fusion.
(g) Which cluster HR diagram or diagrams show the most massive stars?

4. **Doppler velocity curves**
Here are the orbits of two stars in a binary system, along with Doppler velocity curves measured by an observer off to the left and in the plane of the orbits. (The velocity units are not important for this question.)



(a) Which orbit corresponds to the more massive star? How do you know?
(b) Which velocity curve belongs to which star? How do you know?
(c) Consider the points on the velocity curve marked below. Sketch the corresponding locations of the two stars on the orbits. Briefly explain your reasoning.



5. **Jeans' mass and stability**
(a) Assume there is a Giant molecular cloud with a diameter of 20 ly and is at a temperature of 50K with a uniform particle density of $1 \times 10^{-4}\text{cm}^{-3}$. The mean molecular weight of the particles of the cloud is 0.770. Calculate the mass of this cloud in solar masses.
(b) Calculate the total thermal energy (assuming ideal gas) and gravitational energy of the cloud
(c) Will this cloud expand, collapse or remain stable ?
(d) Calculate both Jeans' length (in light years) and Jeans' mass (in solar masses) of the cloud. Are these values consistent with your answer in part (c) ?

6. **Density Profile I**
Suppose a star of local mass M and radius R has a density profile $\rho = \rho_c(1 - \frac{r}{R})$.
(a) Find the total mass M
(b) Solve the pressure profile $P(r)$ with the boundary condition $P(R) = 0$.
(c) Calculate the kinetic energy $K = \frac{3}{2}\int \frac{P(r)}{\rho(r)} dm$ and potential energy. Verify the Virial theorem.

7. **Density Profile II**
Suppose in a stellar model, $\frac{dP}{dr} = -\frac{4\pi}{3} G\rho_c^2 r e^{-\frac{r^2}{a^2}}$ where a is constant.
(a) Find the Pressure $P(r)$ with the boundary condition $P(R) = 0$
(b) Find $M(r)$ and $\rho(r)$
(c) Assuming an ideal gas composition, Show that the temperature $T(r) = T_c(1 - \frac{3}{8}\frac{r^2}{a^2} + \cdots)$

8. **Classical relativistic gas**
For a classical relativistic gas of particles, show that $E_{total} = 0$. As a result, stars dominated by radiation pressure are unstable. The most massive stars that can form are those in which radiation pressure (P_{rad}) and NR kinetic pressure are approximately equal. Estimate the mass of most massive star as follows :
(a) Using gravitational binding energy , show that $P \sim GM^{2/3}\rho^{4/3}$
(b) $P_{rad} \sim P_{kin}$, find the total pressure.
(c) Assuming a fully ionized hydrogen composition , find the maximal mass of star.

9. **Lane-Emden equation for polytropic stars**
(a) Starting with the hydrostatic equilibrium equations for pressure and mass , show that for a polytropic relation between pressure and density $P = K\rho^{1+\frac{1}{n}}$, one can arrive at an equation $\frac{1}{\xi^2}\frac{d}{d\xi}(\xi^2\frac{d\theta}{d\xi}) = -\theta^n$ where where $\rho = \rho_c\theta^n.P = P_c\theta^{n+1}$ and $r = \alpha\xi$. where ξ is a new radius like variable. α is a constant that depends on K, n, G, ρ_c.
(b) The two boundary conditions for the equation are at the center: $\theta = 1, \theta'(\xi) = 0$ at $\xi = 0$. Solve the equation for $n = 0, 1, 5$. The case $n = 0$ corresponds to incompressible fluid. The case $n = 5$ is also special, as the radius of this "star" is infinite. This means that only solutions with $n < 5$ have a surface.
(c) The two cases most interesting for real stars have $n = 1.5$ and $n = 3$, and unfortunately these do not have analytic solutions. Try to solve it numerically.
(d) Show that , for a star of radius R and M , $R^{\frac{3-n}{n}}M^{\frac{n-1}{n}} \approx \frac{K}{G}$. Write down the mass-radius relation for $n = 1.5$ and $n = 3$. Note that , star with $n = 3$ has its mass uniquely determined by the value of K constant, while its radius is not restricted by either K or M.

10. **Dimensional analysis**
(a) For an ideal gas $T \sim \frac{\mu P}{\rho}$. Assuming the star is radiative, Find the dependence on Luminosity on μ, R and M considering the opacity κ varies with ρ and T as $\kappa \sim \rho^{-n}T^{\alpha}$. (For example, for Kramers opacity and an ideal gas EOS $\alpha = 3.5; n = 1$; For Thomson opacity and an ideal gas EOS, $\alpha = 0; n = 0$).
(b) For steadily burning stars, $R(M)$ is obtained by setting the luminosity above equal to the thermonuclear power in the core. If the specific nuclear power (ϵ) varies as $\epsilon \sim \rho^{u-1}T^s$, find the general dependence of L on other variables M, R, u, s. Check that , If we set $s = 20$ (CNO burning), $u = 2, n = 1$ and $\alpha = 3.5$, we find $R \sim M^{0.73}$.